STEADY FLOW AROUND A PLANE CASCADE BY AN IDEAL GAS

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The steady separation-free flow around a flat cascade by an ideal gas is discussed. Most of the attention is devoted to blocking regimes with a supersonic velocity in the entire flow and its subsonic component normal to the front of the cascade. A "directing action" of the cascade (the direction of the velocity and the Mach number of the advancing flow turn out to be related) is exhibited in these regimes which is a consequence of an independence of the flow in front of the cascade of the conditions behind it [1-5]. The most widespread method of their calculation [3, 4, 6] is based on the method of characteristics with establishment of the flow outside the cascade in a timelike coordinate. Although the integrated conservation laws also permit finding the parameters at infinity, the numerical construction of as long-range fields as desired with periodic sequences of attenuating discontinuities is practically impossible. The approximation of nonlinear acoustics (ANA) [7, 8] is justified here, as it is very effective in such problems [8-12]. A combination of ANA, the integrated conservation laws, and establishment in a calculation according to [13, 14] with isolation of the discontinuities has been realized in [5] for the construction of a solution on the entrance section of a cascade and everywhere in front of it. Below the method of [5] is extended to the entire flow and simplified even more. The flow on the entrance section of the cascade is, just as in [3], found in the approximation of a simple wave, in the rest of it and in a finite strip behind it — the flow is found with the help of the "straight-through" version of the scheme of [13, 14], and in the "long-range field" — in the ANA. A simpler version is proposed. In it ANA is applied outside the cascade and the linear theory is applied inside the cascade. Examples of the calculations are given. Similarity laws are formulated for all the regimes of streamline flow.

1. The scheme for flow around a cascade in supersonic closed regimes is illustrated in Fig. 1a, in which xy and sn are rectangular coordinates, the thick lines are shock waves, and the thin ones are the characteristics (the dashed lines are the neutral characteristics going out to infinity). A gas flows from left to right, and in front of and behind the cascade the velocity component normal to its front is subsonic. Therefore as $n \to -\infty$ one should set a single boundary condition, for example, specify the pressure p or, as is done below, one of the Riemann invariants. Depending on its magnitude and the values of the parameters of the advancing flow, the effect of this condition is limited on the left by a discontinuity or a closing c^+-characteristic fe of a bunch of rarefaction waves emerging from f. If ba^o

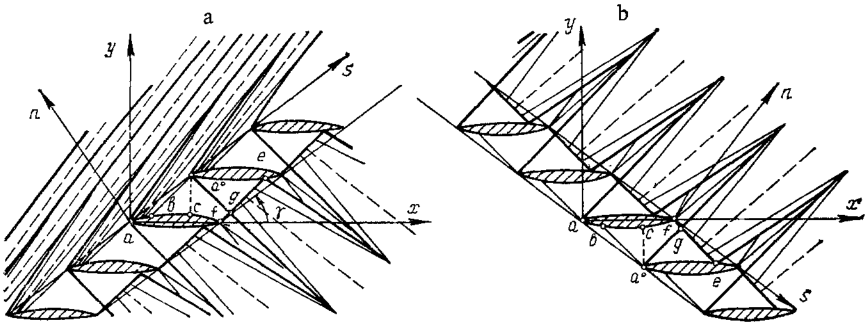

Fig. 1

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is the c^+ characteristic arriving on the leading edge of the upper profile (we are talking about an intervane channel), only the segment ab of the upper generatrix of the lower profile affects the flow in front of the cascade. We shall begin the solution of the problem from a discussion of the flow in the halfplane $n > 0$ and in the triangle aba°; we shall perform an initial analysis of the flow for $n > 0$ by assuming that the parameters are specified at $n = 0$. Then having clarified the structure of the flow for $n > 0$, we shall give a method for its construction and justify the possibility of using a solution of the type of a simple wave in the part of the intervane channel $afga^\circ a$ adjacent to aa°. In the regimes under discussion the shock waves which emerge upward through the flow are usually [5] weak already on the leading edges of the profiles. Therefore the wave structures in front of the leading front can be described within the framework of ANA. Let $\delta\varphi = \delta\varphi(n)$ be the increment to φ on the discontinuity at a distance n from the leading front, and let σ be the specific entropy. Then $\delta\sigma = O(\varepsilon^3)$, where $\varepsilon = \delta p$, the total variation of σ in an infinite sequence of damped discontinuities, is, as has been shown in [11], $O(\varepsilon^2)$, and in front of the cascade

$$\sigma = \sigma_- + O(\varepsilon^2), \quad 2i + V^2 = 2i_- + V_-^2 \equiv 2I_-. \tag{1.1}$$

Here i is the specific enthalpy, $V = |\mathbf{V}|$, and **V** is the velocity vector; a minus sign is affixed to parameters of the advancing flow. The second equality is valid everywhere.

In regions of continuity of the flow on the characteristics

$$dn/ds = \mathrm{tg}(\theta \pm \mu),\ dy/dx = \mathrm{tg}(\beta \pm \mu),\ d\theta \equiv d\beta = \mp[\sqrt{M^2-1}/(\rho V^2)]dp, \tag{1.2}$$

where θ and $\beta = \theta + \gamma$ are angles formed by **V** with the s and x axes, γ is the adjustment angle of the cascade (Fig. 1a, $0 < \gamma \leqslant \pi/2$), $M = V/a$ and $\mu = \arcsin(1/M)$ are the Mach number and angle, a is the speed of sound, and ρ is the density; the upper (lower) sign corresponds to the c^+ (c^-)-characteristic. By virtue of (1.1) and the equations of state $i = i(p, \sigma)$ and $\rho = \rho(p, \sigma)$, the factor in front of dp in (1.2) is, to within an accuracy of $O(\varepsilon^2)$ inclusively, a known function only of p, and one can rewrite the third Eq. (1.2) in the form

$$dJ^\pm \equiv d[\theta \pm \Phi(p)] = O(\varepsilon^2)\,dp, \quad \Phi(p) = \int_{p_*}^{p} \frac{\sqrt{M^2-1}}{\rho V^2}\,dp. \tag{1.3}$$

Here $J^\pm$ are the Riemann invariants; an asterisk is affixed to the critical parameters corresponding to $M = 1$ when $\sigma = \sigma_-$. If the characteristic intersects a discontinuity of the opposite family, the variation of the invariant corresponding to it is, just as also for σ, $O(\varepsilon^3)$, and in this case the total variation of J^- is $O(\varepsilon^2)$. Thence with (1.3) taken into account we find that if $(1 - p/p_-)$ is small (later on this is assumed), when $n \geqslant 0$

$$\theta - \theta_- - \Phi(p) + \Phi(p_-) = O(\varepsilon^2). \tag{1.4}$$

Similarly, if $\varepsilon_0 = \delta p_0 = \delta p(0)$ is the difference in p on the discontinuity at the leading point of the profile and s_0 is the s-coordinate of the point of the line $y = x \tan\gamma$ or $n = 0$ from which the c^+-characteristic in question emerges, on it

$$\theta + \Phi(p) = J^+(s_0) + O(\varepsilon_0^3). \tag{1.5}$$

The residual term in (1.5) is estimated on the assumption that p varies by $O(\varepsilon_0)$ along each c^+-characteristic. Actually, the indicated variation is $O(\varepsilon_0^2)$. This leads to the replacement of $O(\varepsilon_0^3)$ in (1.5) by $O(\varepsilon_0^4)$, which, however, does not change the subsequent estimates.

By virtue of the periodicity in s with the period d, where d is the cascade spacing, we shall consider a single strip resting on a segment of the s axis of length d and bounded by lines which are superposable by a shift along s, for example, adjacent discontinuities or neutral characteristics. According to (1.1), (1.4), and (1.5), the flow in it is, to within an accuracy of ε_0 inclusively, a simple wave with rectilinear c^+-characteristics, each of which (except for the neutral one) is incident on one of the discontinuities. The intensity of a discontinuity is given by the difference in p or J^+, i.e., δJ^+. The righthand side of (1.4), which is related to the increments to σ and J^- upon the intersection of streamlines and the c^--characteristics with an infinite sequence of damped discontinuities, characterizes the variation of the lefthand side of the very same equation in the entire flow, for example, from $n = 0$ to $n \gg d$. For nearby points the righthand sides of (1.4), being $O(\varepsilon^2)$, differ by $O(\varepsilon^3)$. With account taken of what has been said we find from (1.3)-(1.5)

$$\delta p = [\rho V^2/(2\sqrt{M^2-1})]_- \delta J^+ + O(\varepsilon^2), \qquad \delta J^+ = J^+(s^+) - J^+(s^-). \tag{1.6}$$

Here $s^\pm$ are the values of s_0 for the c^+-characteristics which have been incident on a discontinuity for various directions.

Let $\chi(s) = \cot(\theta + \mu)$ be known for $n = 0$. Since the flow under discussion is a simple wave, all the parameters are functions of χ, in particular, $J^+ = J^+(\chi)$, where $\chi(s)$ is a periodic function of period d which is discontinuous in the general case when $s = kd$ ($k = 0, \pm 1, \ldots$). The breaks are caused by discontinuities on the leading edges of the profiles. With a known $\chi(s)$ the construction of the discontinuity reduces to the solution of the equations [8]

$$S = s^- + n\chi(s^-) + O(\varepsilon_0^2),\ S = s^+ + n\chi(s^+) + O(\varepsilon_0^2),$$
$$dS/dn = [\chi(s^-) + \chi(s^+)]/2 + O(\varepsilon^2) \tag{1.7}$$

with estimates of the errors ($s = S(n)$ is the equation of the discontinuity) which are uniform in n. When $S(0) = 0$, Eqs. (1.7) determine S and $s^\pm$ as functions of n. Then one can find the intensity of a discontinuity from $s^\pm$ with the help of (1.6). After this (see below) the parameters of the advancing flow are determined from the distributions on aa°, and the flow between adjacent discontinuities is found from (1.1), (1.4), and (1.5) and the equation of almost rectilinear c^+-characteristics

$$s = s_0 + n\chi(s_0) + O(\varepsilon_0^2). \tag{1.8}$$

In the general case the system (1.7) is solved numerically; it is convenient to use instead of the first two Eqs. (1.7) the result of their differentiation with respect to n and the elimination of dS/dn. The resulting equations are of the form

$$ds^\pm/dn = (\chi^\mp - \chi^\pm)[2(1 + n\chi_s^\pm)]^{-1} + O(\varepsilon^2), \tag{1.9}$$

where $\chi^\pm = \chi(s^\pm)$ and $\chi_s = d\chi(s_0)/ds_0$. In the case under discussion $\chi_s > 0$.

With a linear dependence of χ on s_0 the system (1.9) is integrated. Actually, subtracting the second (with the lower subscripts) Eq. (1.9) from the first one and multiplying the result by $\chi_s \equiv -\delta\chi_0/d$, we obtain the equation

$$d(\delta\chi)/d\nu = (1 + \nu\delta\chi_0)^{-1}\delta\chi_0 + O(\varepsilon^3)\ (\nu = n/d).$$

Having integrated it from $\nu = 0$, where $\delta\chi = \delta\chi_0$, and switching from $\delta\chi$ to δp, we find that in this case

$$\frac{\delta p}{\delta p_0} = \left[1 + \frac{\nu\alpha\delta p_0 \operatorname{tg}\mu}{2\rho a^2 \sin^2(\theta + \mu)}\right]^{-1} + O(\varepsilon_0^2) \quad (\alpha = \rho^3 a^4 \omega_{pp}), \tag{1.10}$$

where $\omega = 1/\rho$, $\varphi_{pp} = (\partial^2\varphi/\partial p^2)\sigma$, and one can calculate the quantities without subscripts (except δp) from the parameters as $n \to \infty$. For a perfect gas $\alpha = 1 + \varkappa$, where $\varkappa$ is the adiabatic exponent, and (1.10) reduces to the well-known formulas of [5, 10, 11]. For any nonlinear distribution $\chi(s_0)$ the distribution of χ over s becomes linear as one moves away from the cascade. Therefore if ν is the distance from the corresponding cross section and δp_0 is the intensity of the discontinuity in it, (1.10) gives the long-range field for any distributions on aa°. According to the calculations of [5], in typical situations a linear dependence of χ on s is established in 3-4 strips on n.

The condition of conservation of J^+ on the c^+-characteristics, which together with (1.1), (1.4), and (1.8) describes the flow between discontinuities, is equivalent to the equation

$$\partial J^+/\partial n + \chi\partial J^+/\partial s = 0.$$

Since $J^+ = J^+(\chi)$ with $J_\chi^+ \equiv dJ^+/d\chi = (\chi_J)^{-1} \neq 0$, having divided it by J_χ^+, we obtain

$$\partial\chi/\partial n + \chi\partial\chi/\partial s = 0. \tag{1.11}$$

If Γ is a closed contour of the sn-plane, (1.11) and the last Eq. (1.7), which determines the direction of the discontinuity, follow from the "integrated conservation law"

$$\oint_\Gamma \chi ds - \frac{\chi^2}{2} dn = 0. \tag{1.12}$$

The "area rule" [7, 8], which permits constructing discontinuous solutions without numerical integration of (1.7) and (1.9), is valid for (1.12).

In any method of constructing a solution in front of a cascade the parameters with a minus subscript, which correspond to a uniform advancing flow (theoretically as $n \to \infty$) are found, just as in [3-5], using the distributions at $n = 0$ from the integrated conservation laws of mass, momentum, and energy written for a closed contour formed by adjacent discontinuities and segments of the s axis of length $\Delta(s/d) \equiv \Delta\zeta = 1$ for $n = 0$ and $n \to \infty$. At $n = 0$ the parameters of the flow satisfy, as everywhere, the condition $2i_0 + V_0^2 = 2I_0$ with a constant I_0 and with $\varphi_0 = \varphi_0(\zeta) \equiv \varphi(\zeta, 0)$. With this taken into account the enumerated laws take the form

$$\mathbf{R}_- = \int_0^1 \mathbf{R}_0(\zeta)\, d\zeta, \quad 2i_- + V_-^2 = 2I_0, \tag{1.13}$$

where R is a vector with the components $\rho V \sin\theta$, $\rho V^2 \sin 2\theta$, and $p + \rho V^2 \sin^2\theta$.

With known righthand sides the system (1.13) along with the equation of state $i = i(p, \rho)$ uniquely determines the supersonic ($M_- \geqslant 1$) advancing flow, in particular, σ_- and J_-^-. In the problem under discussion the discontinuities form small angles with the c^+-characteristics. Therefore the intensity of each discontinuity on a length of the order of d varies by $O(\varepsilon^2)$, and the overall nonuniformity in σ and J^- as aa° obtained due to the action of all the discontinuities is $O(\varepsilon_0^3)$. This is in agreement with the fact that $\delta\sigma_0$ and δJ_0^- are also $O(\varepsilon_0^3)$. And so one can replace the nonuniform distributions of σ and J^- at n = 0 to within any accuracy of ε_0^2 inclusively with constants σ_0 and J_0^- which differ from σ_- and J_-^- by $O(\varepsilon_0^2)$. The increase of σ characterizes the irreversible losses in the discontinuities ($\sigma_0 > \sigma_-$). The error, smaller than in the entire semi-infinite strip, of a solution of the simple wave type on the entrance section $afga^\circ a$ of the intervane channel is a consequence of its finite (of the order of d) dimensions.

What has been said justifies the partial replacement of the direct problem by the inverse one. In the latter instead of σ_- and J_-^-, σ_0 and J_0^- are fixed, whose specification along with $I_0 \equiv I_-$ permits constructing in the simple wave approximation a flow in $afga^\circ a$. Precisely this approach has been adopted in [3], in which it is true that not J_0^- but the point b was specified. The parameters of the advancing flow (as $n \to \infty$) are determined from the distributions found from (1.13) on aa°, and the wave structure which arises in front of the cascade is constructed within the framework of the ANA. Since the advancing flow is characterized by four parameters, let us assume, p_-, ρ_-, M_-, and θ_- or β_-, and the solution depends only on three constants I_0, σ_0, and J_0^-, one of the parameters, for example θ_-, turns out to be a function of the other three. This means that in blocking regimes the cascade exerts a directing action on the flow in front of it. For a perfect gas I_0 and σ_0 only give the velocity, pressure, and density scales for a fixed $\varkappa$ and a specified cascade $\theta_- = f(M_-)$. The indicated property (see [1-4]) is not related to the simplifications introduced into the method of solution.

Two methods were used to construct the flow in the remaining part of the intervane channel. Numerical solution on layers x = const using the straight-through version of the difference scheme of [13, 14] lies at the basis of the first method. The segment ca°, but not the shock waves or the characteristics $a^\circ g$ and fg, which are not known in advance, was taken as the cross section of the initial data for simplification of the numerical algorithm. The second method was based on linearization of the equations of the characteristics (1.2). The linearization was performed with respect to a supersonic translational flow with parameters equal to the parameters on ba°, to which the subscript "b" is affixed. After linearization all the characteristics of the same kind have the identical slope, and the parameters are discontinuous on those which replace compaction discontinuities or rarefaction bunches. The linearized invariants introduced in accordance with (1.2) by one of the two methods:

$$J^\pm = \beta \pm Bp, \quad J^\pm = \beta \pm B^0\pi, \quad B = \sqrt{M_b^2 - 1}\big/(\rho V^2)_b, \quad B^0 = Bp_b \tag{1.14}$$

are conserved along the characteristics (including at an intersection of discontinuities — the characteristics of the opposite family). The first method corresponds to ordinary linearization. The second one differs in the replacement of p by $\pi = \ln p$, which, as is well known [15], reduces the errors of the linear theory at moderate supersonic velocities. The other equations (of constant entropy, constant energy, state, and so on) as well as the boundary condition $\beta = \beta(x)$ on the profiles were not linearized. Although such refinements are out of order in "order of magnitude" estimates, in practice they always raise the accuracy of the results.

Calculations using the constancy of the invariants (1.14) on segments of characteristics are performed using finite formulas and reduce to the sequential determination of parameters on the profiles and in specified cross sections. For thin profiles the use of the linear theory inside the cascade (including on the entire entrance section) does not contradict the necessity of drawing on ANA outside of it; the nonlinear effects are of a cumulative nature building up at large distances.

For convenience in the subsequent exposition we shall change the orientation of the cascade and the y and n axes so that they are arranged as shown in Fig. 1b. The points a, b,

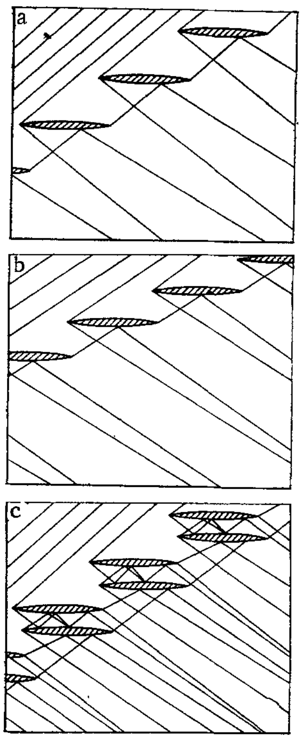

Fig. 2

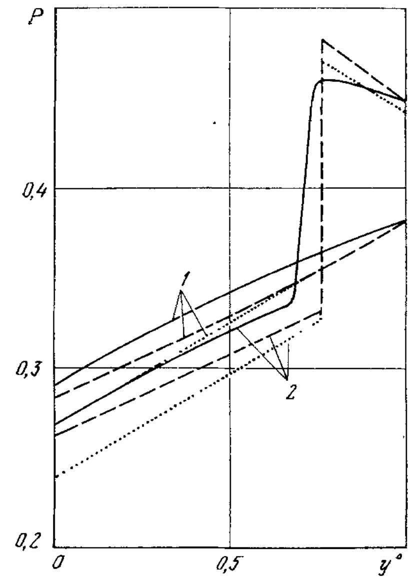

Fig. 3

... and the direction of the x and s axes coincide in Fig. 1a and b, and J^+ and J^- exchange roles. As has already been noted, the conditions downstream affect the section of the intervane channel to the right of fe. In Fig. 1b their influence is brought about by specification of J^- as $n \to \infty$. It is simpler here to invert the problem partially by specifying not J_+^- but J_0^-. If the initial (at n = 0) intensity of the discontinuities emerging downstream is comparatively large and it is impossible to apply the linear theory and near n = 0 the ANA, the flow behind the cascade in the strip $0 \leqslant n \leqslant n_0$ is calculated by the method of [13, 14] with establishment in x. J^- is specified on the streamline coming out of f, and three cascade spacings are sufficient for establishment in typical examples. Usually a region which occupies only barely more than a single spacing is calculated by the same method in a calculation on layers in the inverse formulation. This is done in order to form initial distributions on the line $n = n_0 > 0$ for ANA and the determination (using (1.13) with a plus subscript instead of a minus) of the parameters far downstream. In the linear approach the required distributions are obtained on the trailing front.

Among the distinguishing characteristics of the flow behind the cascade one should refer first of all to the fact that the entropy σ_+ exceeds σ_0 due to losses in the discontinuities. Secondly, behind the cascade weak discontinuities of the same family usually intersect. However, this does not complicate the analysis, since in ANA such intersections lead to a merging of discontinuities without the appearance of singularities of the other family. To sum up, formula (1.10) is true far from the trailing front as before.

2. The approaches described in Sec. 1 were implemented on a BÉSM-6 computer in programs adapted for calculation of the flow around single and so-called biplane cascades. The possibilities of these programs are demonstrated by the examples presented in Fig. 2. The discontinuities which arise in connection with streamline flow by a perfect gas with $\varkappa = 1.4$ are plotted as thick lines in Fig. 2 for two single (Figs. 2a and b) and one biplane (Fig. 2c) cascade, the symmetric doubly comvex profiles of which are formed by arcs of circles of radius $r/d = 5$ with a chord length $l/d = 1$. The biplane cascade (Fig. 2c) is formed by two cascades of Fig. 2a shifted slightly ahead one of the other. The flow behind all the cascades in Fig. 2 was determined by specifying the right invariant on its trailing front: $J^+ = 2.08$. In the case of Fig. 2a and c $\gamma = 30°$, and in the case of Fig. 2b $\gamma = 20°$. The calculated regimes were characterized by the following values of the Mach number and the angle β of the advancing flow: Fig. 2a, c) $M_- = 1.7$, $\beta_- = 4.15°$; Fig. 2b) $M_- = 2$, $\beta_- = 3.33°$. The ratio $\Sigma \equiv P_+/P_-$, where P is the retarding pressure, is somewhat higher for the biplane cascade (0.932) than for a single cascade (0.926). Comparing these values, one should bear in mind that p_+/p_- also differ in the calculated cases. The straight-through computation method was used

inside the cascade (to the right of ca^o) to obtain the results presented in Fig. 2. Calculation of a single cascade with the number of cells across the channel N = 30, which provides for a constancy (upon the further increase of N) of no less than three significant digits in Σ and in the parameters as $n \to \pm\infty$, requires 2 min on a BÉSM-6. The analogous calculation of a biplane cascade with N = 80 in its two channels requires no more than 6 min. In the linear approximation these alternatives are considered with completely acceptable accuracy. This confirms Fig. 3, in which in the typical case the distributions of p found by three methods are plotted for two cross sections ($x = x_c$ — curves 1, and $x = x_f$ — curves 2): solid curves — by numerical integration according to [13, 14], dashed curves — by the linear theory with replacement of p by π, and dotted curves — without a replacement (y^o is the distance from the lower profile relative to the channel height in a given cross section). The transition from p to π reduces the errors of the linear theory.

The approaches and programs developed are applicable also in cases in which $M_n \equiv |V\sin\theta|/a > 1$ and the flow in front of the cascade front is not disturbed as well as in similar problems on supersonic outflow from cascades of flat nozzles. If $M_n > 1$ behind the cascade, the flow in general case contains discontinuities of both families. After the discontinuities become weak, their subsequent damping is described, as in [9, 11], by ANA, the waves of the different families are discussed independently, and the overall distributions are obtained by their superposition.

3. Without restricting ourselves later to regimes with M > 1, we shall switch to the similarity laws for steady flow around cascades. In the case of thin profiles and values of M not close to unity their derivation is based on linearization of the equations and boundary conditions. It is sufficient for a cascade to consider the strip $-\infty \leqslant x \leqslant \infty$, $0 \leqslant y \leqslant y_{a^o} = d \sin\gamma$, setting up in addition to the condition of no through-flow on the profiles the periodicity condition $\varphi(x + x_{ao}, y_{ao}) = \varphi(x, 0)$ at $x < 0$ and $x > l$, where φ is some parameter and $x_{ao} = d\cos\gamma$. We shall restrict ourselves to affinely similar profiles: $y = \tau/\Gamma_\pm(x^o)$, where $x^o = x/l$, τ is the relative deviation of the generatrices from the chord, and $\Gamma_\pm(x^o)$ are functions of the order of unity; a plus (minus) sign gives the upper (lower) generatrix of the profile located on the x axis; $\Gamma_+(0) = \Gamma_-(0)$ and $\Gamma_+(1) = \Gamma_-(1)$. For a streamline flow with $M_- > 1$ and $M_{n-} < 1$ the similarity law obtained by the method described above reduces to the equations

$$\begin{gathered} u' = V_-\lambda^{-1}\tau u^0, \quad v = V_-\tau v^0, \quad p' = -\rho_- V_-^2\lambda^{-1}\tau u^0, \\ \rho' = a_-^{-2}p', \quad \varphi^0 = \varphi^0(x^0, y^0, \xi, \eta, j), \\ X = \rho_- V_-^2\lambda^{-1}\tau^2 l X^0, \quad Y = \rho_- V_-^2\lambda^{-1}\tau l Y^0, \\ \Phi^0 = \Phi^0(\xi, \eta, j), \quad \lambda = \sqrt{M_-^2 - 1}, \quad y^0 = y\lambda/l, \\ \xi = x_{a0}/l, \quad \eta = y_{a0}\lambda/l, \quad j = J_+^+/\tau, \end{gathered} \tag{3.1}$$

where u, v and X, Y are the projections of V and the force acting on the profile onto the x and y axes and the notation $u = u_- + u'$, ..., is adopted for u, p, and ρ; in this approximation $u_- = V_-$. In blocking regimes the effect of the parameter j is bounded on the left by a discontinuity or the characteristic fe. In unblocked regimes with $M_- > 1$, when the indicated discontinuity emerges upstream (this occurs for sufficiently widely spaced cascades), j affects the entire flow. However, there is no directing action, and one can take $\vartheta = \beta_-/\tau$ instead of j as the similarity parameter. By virtue of (3.1) as well as the linearized conservation laws (1.13), we have in blocked regimes with distributions on the leading front which satisfy (3.1)

$$\vartheta \equiv \beta_-/\tau \simeq v_-/(V_-\tau) = \varphi(\xi, \eta). \tag{3.2}$$

In order to check (3.1) and (3.2), calculations of the flow around six cascades with $\xi = \sqrt{3}/2$ for different M_- were made with the help of the approach described in Sec. 1 without using the linearized relationships (1.14). Their results are given in Fig. 4 (a — dependence of β_- in degrees on M_-; b — dependence of ϑ on η). Curves corresponding to cascades with the following values of γ, d/l, and τ are marked with numbers: 1) 16.1°, 0.9015, 0.025; 2) 30°, 1, 0.025; 3) 45°, $\sqrt{3}/2$, 0.025; 4) 16.1°, 0.09015, 0.0125; 5) 30°, 1, 0.0125; and 6) 45°, $\sqrt{3}/2$, 0.0125. It is evident that (3.2) groups the curves pertaining to different cascades within limits not exceeding ±10% of ϑ.

The similarity law (3.1), (3.2) obtained in the linear approximation provides no similarity of nonlinear damping of the wave structures far from the cascade. As one can show

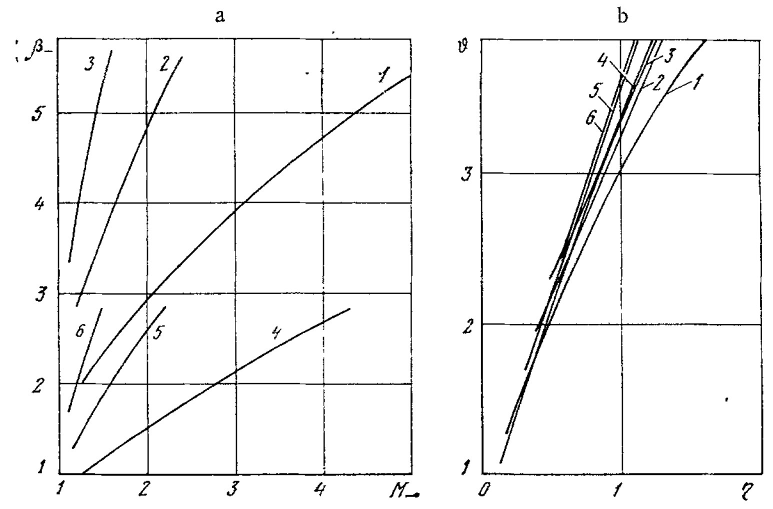

Fig. 4

in ANA, this requires constancy of $K_- = \lambda^2(M_-^4\alpha\tau)^{-2/3}$, where α is the same as in (1.10), in addition to the similarity parameters from (3.1). This condition is obtained from an analysis of the equation in ANA which determines the slope of the rectilinear characteristics in the x^oy^o plane. In contrast to (3.1), constants which characterize the thermodynamics of the medium enter K_- through α. For a perfect gas $\alpha = 1 + \varkappa$ and $K_- = \lambda^2[M_-^4(1+\varkappa)\tau]^{-2/3}$. In supersonic regimes with $M_{n-} > 1$, in which the flow in front of the cascade is not disturbed, it is convenient to replace j in (3.1) by ϑ, as in unblocked regimes with $M_- > 1$. The very same replacement is necessary (and not just convenient) in the case of a completely subsonic streamline flow. In addition $\lambda = \sqrt{1 - M_-^2}$ here. The corresponding similarity law which generalizes the Prandtl–Glauert similarity law for a single profile is well known [16]. We note that due to the d'Alembert paradox $|X^o| = \vartheta|Y^o|$.

Now let a transonic flow occur around a cascade. Proceeding in the same way here as in [17], we find that in unblocked regimes

$$\begin{gathered}
u = a_*\left(1 + \tau^{2/3}\alpha^{-1/3}u^0\right), \quad v = a_*\tau v^0, \quad p = p_* - \rho_* a_*^2\tau^{2/3}\alpha^{-1/3}u^0, \\
\rho = \rho_*\left(1 - \tau^{2/3}\alpha^{-1/3}u^0\right), \quad \varphi^0 = \varphi^0(x^0, y^0, \xi, \eta, K, \vartheta), \\
X = \rho_* a_*^2\tau^{5/3}lX^0, \quad Y = \rho_* a_*^2\tau^{2/3}lY^0, \Phi^0 = \Phi^0(\xi, \eta, K, \vartheta), \\
x^0 = x/l, \quad y^0 = y(\tau\alpha)^{1/3}/l, \quad \xi = x_{a0}/l, \quad \eta = y_{a0}(\tau\alpha)^{1/3}/l, \\
K = (M_- - 1)(\tau\alpha)^{-2/3}, \ \vartheta = \beta_-/\tau
\end{gathered} \tag{3.3}$$

with α from Sec. 1 (one of the expressions for α in [17] is not true).

If the flow in front of the cascade is supersonic and $M_n < 1$, which is usually satisfied in transonic regimes for $\gamma < \pi/2$, (3.3) provides similarity of the short-range and long-range fields, as one can show. In blocking regimes ϑ becomes a function of ξ, η, and K, and X^o, Y^o, and the parameters behind the cascade (to the right of the so-called closing discontinuity for $M_- < 1$ and of fe in the opposite case) become functions not of ϑ but of $\pi = (p_+ - p_*)\alpha^{1/2}/(\rho_* a_* \tau^{2/3})$ with p_+ specified. In a supersonic outflow from a cascade with $M_n < 1$ one can replace the parameter π by j from (3.1). Although approaches different from those developed in Sec. 1 (see [18]) are required for calculation of transonic regimes, the supersonic long-range fields are also described here by ANA.

The similarity laws (3.1)-(3.3) are based on the assumption of smallness of the perturbations, which is frequently violated in some local regions. The neighborhoods of blunt leading edges, and in the cases of sub- and transonic streamline flow — the critical points (including the trailing edges around which flow occurs according to the Chaplygin–Zhukovskii scheme) and leading sharp edges, are the same in all regimes. Here (3.1) and (3.3) naturally do not occur. Moreover, even if such singularities have no effect on the fields of parameters far from them, their action on the integrated characteristics may be noticeable (for example, due to an inflow force on the leading edge), causing deviations of X and Y from (3.1) and (3.3).

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LITERATURE CITED

1. G. Yu. Stepanov, The Hydrodynamics of Cascades of Turbomachines [in Russian], Fizmatgiz, Moscow (1962).
2. G. L. Grodzovskii, A. A. Nikol'skii, G. P. Svishchev, and G. I. Taganov, Supersonic Flows of a Gas in Perforated Boundaries [in Russian], Mashinostroenie, Moscow (1967).
3. O. K. Lawaczeck, "Calculation of the flow properties up- and downstream of and within a supersonic turbine cascade," Pap. ASME, 1972, N GT-47.
4. H. J. Lichtfuss and H. Starken, "Supersonic cascade flow," in: Progress Aerospace Sci., Vol. 15, Pergamon Press, Oxford (1974).
5. A. B. Borod, A. N. Kraiko, and E. Ya. Chernyak, "An investigation of flow around a flat cascade by a supersonic flow of an ideal gas with a subsonic "normal" component in regimes with associated discontinuities," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 4 (1979).
6. R. E. York and H. S. Woodard, "Supersonic compressor cascades. An analysis of the entrance region flow field containing detached shock waves," Trans. ASME, Ser. A, J. Eng. Power, 98, No. 2 (1976).
7. L. D. Landau and E. M. Lifshits, The Mechanics of Continuous Media [in Russian], Gostekhizdat, Moscow (1953).
8. J. Wizem, Linear and Nonlinear Waves [Russian translation], Mir, Moscow (1977).
9. G. A. Simons, "Decay of a diamond shock pattern," AIAA J., 10, No. 8 (1972).
10. M. R. Fink, "Shock wave behavior in transonic compressor noise generation," Pap. ASME, 1971, N GT-7.
11. A. N. Kraiko and A. A. Osipov, "Damping of a periodic sequence of weak shock waves in channels with 'sound-absorbing' walls," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 4 (1976).
12. A. N. Kraiko and A. L. Ni, "The approximation of nonlinear acoustics in problems of oscillations of a gas in tubes," Prikl. Mat. Mekh., 44, No. 1 (1980).
13. M. Ya. Ivanov, A. N. Kraiko, and N. V. Mikhailov, "The method of straight-through calculation for two-dimensional and three-dimensional supersonic flows. I," Zh. Vychisl. Mat. Mat. Fiz., 12, No. 2 (1972).
14. S. K. Godunov, A. V. Zabrodin, M. Ya. Ivanov, et al., Numerical Solution of Multidimensional Problems of Gas Dynamics [Russian translation], Mir, Moscow (1976).
15. A. N. Kraiko and R. A. Tkalenko, "The construction of a linear theory of nonequilibrium and equilibrium flows," Izv. Akad. Nauk SSSR, Mekh. Zhick. Gaza, No. 6 (1968).
16. K. Oswatitsch, Foundations of Gas Dynamics [in German], Springer-Verlag, Vienna-New York (1976).
17. A. N. Kraiko and R. K. Tagirov, "Near-sonic flow around bodies of revolution with a channel in the presence of a jet flowing out of the channel," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6 (1974).
18. A. F. Messiter and T. C. Adamson, Jr., "Transonic small-disturbance theory for lightly loaded cascades," AIAA J., 19, No. 8 (1981).